CHAPTER 7

A Model of Microinsurance and Reinsurance

Stéphane Bonnevay, David M. Dror, Gérard Duru, and Michel Lamure

Reinsurance offers insurance companies many advantages, including stabilization of losses and surplus enhancement, according to Outreville (chapter 3, this volume). Can reinsurance therefore ensure the financial stabilization of multiple microinsurance units? This question can be approached in two ways: empirically or theoretically.

The *empirical approach* consists of carrying out repeated field studies in real-life settings and observing the results. However, since there are no field data on reinsurance transactions with microinsurers, whether reinsurance can work for small health schemes needs to be assessed through theoretical reasoning. The *theoretical approach* is based on identifying qualitative information concerning possible solutions, through simplified representations of the problem *(the model),* and then defining a calculation protocol to validate (or invalidate) the model's underlying hypothesis through the results (Lesage 1999). This chapter describes the theoretical approach followed in the model.

This model tests the hypothesis that microinsurance schemes, operating on their own, are financially less viable than they would be if they pooled their risks through reinsurance. This proof entails consideration of three subissues:

- Demonstration of the positive effect of reinsurance on microinsurers' financial viability
- Exploration of the utility of reinsurance for microinsurers and the variables affecting their decision to reinsure
- Elaboration of a protocol for calculating the reinsurance premium, based on analysis of scenarios that are likely to occur in reality

THE PROBLEM

Reinsurance involves two sets of contracts, between each microinsurer and its members and between each microinsurer and its reinsurer.

For their invaluable contributions to this chapter, the authors thank Michael Cichon, Chief, Financial, Actuarial and Statistical Branch, Social Protection Sector, International Labour Office; Ivan Lavallée, Professor of Informatics, University of Paris 8; and Ruth Koren, Professor, Faculty of Medicine, Tel Aviv University.

The Contract between Microinsurer and Its Members

Under the basic contract between each microinsurer and its members, each member pays the microinsurer a periodic contribution. In return, the microinsurer agrees to pay specified medical costs for the insured. The viability of the basic contract depends on the microinsurer's ability to pay its obligations in full at any future time. Because its ability to do so depends on its financial stability, the microinsurer's profit (or loss) at the end of each accounting period has to be projected. To make these projections, parameters are needed for determining the financial outcome.

Anticipating each microinsurer's business results depends on the number of times events covered by the insurance contract occur and on the costs associated with these events. Both variables fluctuate randomly.

For example, a microinsurer may agree to cover members for up to five days' hospitalization. When the insurance policy is signed, how often each individual will be hospitalized during the term of the contract is unknown. However, within a given target population, the probability distribution can be estimated for the random variable: "number of hospitalizations of up to five days during a certain period." The cost of one event of hospitalization is not constant, because the length of stay and the cost per day may vary. Nonetheless, if the distribution functions of both unit cost and incidence are available, the probability distribution of the overall cost for this type of event can be deduced for the entire microinsurance unit over a given period. The same process can be applied to all benefits included in the insurance package.

The balance at the end of an accounting period is therefore random. How, then, can the probability distribution of this balance be determined? As the balance reflects a difference between income and expenditure, its probability distribution depends on income-side values (including members' contributions, external resources funneled into the microinsurance unit, and the microinsurer's available reserves) and on expenditure-side values (including administrative costs and the composition of the benefit package), and the number of members covered by the microinsurer. With knowledge of the distribution of the microinsurer's business results, its chances of bankruptcy or survival can be estimated.

The microinsurer's financial stability will be measured here in terms of its risk of becoming insolvent (*failure rate*), as the membership has a self-explanatory interest in eliminating or reducing the risk of insolvency. The hypothesis (as defined earlier) can therefore be rephrased to state that transferring part of the risk to reinsurance reduces the microinsurer's failure rate.

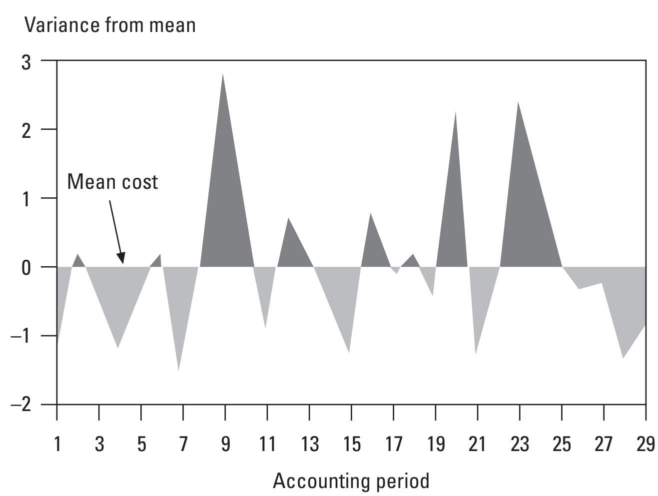
The Contract between Each Microinsurer and Its Reinsurer

Under the basic contract between each microinsurer and its reinsurer, the microinsurer pays the reinsurer a periodic premium. In return, the reinsurer pays the microinsurer for costs exceeding a specified *reinsurance threshold*. The fundamental assumption here is that the client microinsurers' business results can fluctuate around a mean value. Thus, in "good years" (when costs are below the mean), the microinsurer will run a surplus (compared with the mean), and in "bad years" (when the costs exceed the mean) the microinsurer will run a deficit (compared with the mean). These fluctuations stem, to a great extent, from the microinsurers' small membership and claim load. The extent of fluctuations can be estimated by applying statistical laws, if the probability of events and their average cost are known. However, no one can know whether the surplus in good years will cover the deficit in bad years, or that good years will precede bad ones, such that the microinsurer always has enough reserves to cover deficits. Therefore, if the microinsurer wishes to lower its financial risk exposure to the mean cost, which is much more predictable and affordable, it has to obtain an alternative source to cover these mean costs. This is what the reinsurance offers to do (figure 7.1).

Reinsurance offers the microinsurer a dual advantage. First, it avoids the risk of bankruptcy in bad years. Second, by freeing the microinsurer from unexpected fluctuations in expenses, reinsurance also removes the microinsurer's obligation to maintain contingency reserves and enables it to use surpluses generated in good years at its own discretion.

This relationship is based on the assumption that the reinsurer remains solvent at all times. Yet, the reinsurer also faces a risk of bankruptcy, as its business results, too, are determined by income (premiums collected) and expenditures (benefits payable to microinsurers plus administrative costs). Therefore, the reinsurer's probability of bankruptcy has to be calculated at the end of each period. The reinsurer's estimated risk of insolvency depends on the probability distribution of client microinsurers' business results, the contract terms, and

FIGURE 7.1 The Dual Advantage of Reinsurance



administrative costs. Additionally, the number of reinsured microinsurers (the size of the pool) may influence the reinsurer's business results.

This set of parameters will be structured into a model, described below.

THE PRINCIPLES UNDERLYING THE REINSURANCE MODEL

The reinsurance model proposed here is based on two major principles.[1] First, as the reinsurer's role is to shield its client microinsurers from the risk of insolvency arising out of the cost of random fluctuation of insurable events, the reinsurer takes account only of the microinsurer's insurable activities (for a distinction between insurable and uninsurable events see Vaté and Dror, chapter 6, this volume). Second, in exchange for a premium, reinsurance covers the microinsurer's costs that exceed the reinsurance threshold.

For each insurable activity at the beginning of each accounting period, the microinsurer estimates the resources it will need to pay the benefits. This amount is based on the microinsurer's forecasts of its income and its exposure to cost-generating events during the period. As the actual cost may vary from the estimated cost because of random fluctuations, the exact amount will be known only at the end of a period when the microinsurer finds out whether it has incurred a profit or a deficit. In the model, a deficit is considered a bankruptcy situation (because of the underlying assumption that microinsurers do not maintain contingency reserves to cover fluctuations in the cost of their insurance activity). In the model, the reinsurer covers deficits resulting from higher-than-expected expenses. At the outset, their calculation requires a definition of the level of benefits payable by the microinsurer to the members before the reinsurer takes over (*unceded risk*, called here the *reinsurance threshold),* and the costs for which the reinsurer is responsible *(ceded risk)*. The contract (*treaty)* also sets the premium microinsurers must pay for reinsurance.

The two principles mentioned above are necessary to model a comparison of the microinsurer's failure probabilities with or without reinsurance. In addition, the model allows identification of the minimum resources that each microinsurer must secure to cover its unceded risk and pay the reinsurance premium.

FORMULATION OF THE MODEL

The mathematical formulation of the relationship between a reinsurer and microinsurers is elaborated in annex 7A.

The model is formulated to answer the fundamental question: When will a microinsurer want to purchase reinsurance? The reply seems to be: When the reinsurance contract reduces the resources needed to secure at least the same level of solvency for a defined level of expenditure (linked to a defined benefit package). This calls for a comparison between two quantities, which would be straightforward if both amounts were known at the same time. However, the values determining the microinsurer's expenses (for example, number of events, unit cost, length of hospital stay) are random and can fluctuate within a certain range according to the distribution probability. Therefore, the microinsurer's precise payout liability for the entire accounting period is unknown in advance. On the other hand, the maximum cost can be estimated if the distribution is known (as is assumed under the model). The level of solvency also has to be determined, bearing in mind that 100 percent survival (without reinsurance) can be guaranteed only when resources to cover the worst-case scenario are available at the beginning of the period. An example for this comparison is a situation where a microinsurer without reinsurance needs resources equal to its mean benefits, plus a safety margin proportional to the variance of its benefits.[2] Assuming that the reinsurance threshold is equal to the mean benefits, reinsurance would be advantageous for the microinsurer if the reinsurance premium were cheaper than the safety margin. This comparison is pertinent in so far as both options secure the same level of solvency.

When reinsurance is considered, one of the two amounts (the cost of the reinsurance premium plus the microinsurer's reinsurance threshold) ceases to be an estimate and is defined in the reinsurance treaty. The other amount is an estimate of the maximum capitalization needed to guarantee full self-insurance. In reality, as expenditures fluctuate, some years the microinsurer will need less than the maximum, and the challenge is to operate with as little capital as possible at the beginning of the period without increasing the failure rate.

The reinsurance premium has to cover the reinsurer's solvency, which, like that of the microinsurer, is defined up front. A lower solvency rate of the reinsurer would translate into a lower premium, but such a reinsurer may fail to provide adequate levels of solvency for the microinsurer. Hence, in all the examples elaborated here, the reinsurer's solvency rate is assumed to be 95 percent. The conditions that satisfy this requirement will depend on the number of microinsurers in the pool and on each pooled microinsurer's risk profile. An example of an analytical calculation of the reinsurance, applying the model (but for a simplified scenario of 30 microinsurers[3] with identical mean and variance and a simple statistical distribution law for the benefit cost) is illustrated below, based on equations elaborated in annex 7B.

Let us consider a simplified example where more than 30 identical microinsurers sign an identical reinsurance contract for one time period. A uniform distribution of benefits payable by microinsurer to members is assumed,[4] on an interval of [0, 10] monetary units (₪).[5] In this case, without reinsurance, the microinsurer needs 10 ₪ at the beginning of each period to ensure its solvency. With reinsurance, the microinsurer would need to secure, in addition to the premium, only the mean cost, in this case 5 ₪, the reinsurance threshold, because the reinsurer bears all costs above it.

As an illustration, two groups (one with pool size of 36 microinsurers and the other with pool size of 100 microinsurers) are compared, and two scenarios are presented for each group. Under *Scenario 1*, the reinsurer's initial capital is equal to the administrative costs, and all other expenses are covered only from premium income. Under *Scenario 2*, the reinsurer's initial capital is equal to the administrative costs plus 0.5 ₪ per microinsurer (the equations and details of the calculations are provided in annex 7B).

As can be seen in table 7.1, in any of the four permutations, the microinsurers would need less than 7 ₪ to ensure 100 percent survival with reinsurance, compared with 10 ₪ without reinsurance. Put another way, with the same resources, the microinsurers would incur a risk of insolvency ranging from 48 percent to 33 percent without reinsurance, but 0 percent with reinsurance. The conclusion is that in this example, for any level of resources, reinsurance reduces microinsurers' failure rate. Furthermore, for 0 percent failure rate, the level of resources needed is reduced considerably.

Now we have to recognize that, in the example discussed so far and in annex 7B, an analytical approach could be followed, but to do so, some assumptions had to be simplified. However, the mean benefit cost for a microinsurer and the variance of this parameter can be calculated under more realistic scenarios even when their statistical distribution function is not known, by making an assumption regarding the statistical distribution of the incidence of benefits (for instance Poisson distribution law[6]) and of unit cost (Chi-squared distribution law[7]). The method for this calculation is elaborated in annex 7C.

This calculation requires an estimate of the probability of each benefit and its mean unit cost. As in other cases of statistical sampling, the better the data, the more reliable the extrapolated figures will be. The data for these calculations originate from the microinsurers themselves, and may initially be somewhat weak. It is hoped that the quality of such estimates will improve when data are collected over a longer period of time and in line with a methodology proposed by the *Social Re Data Template*.[8]

However, even when the mean benefit cost of each microinsurance unit is known and the variance in this value can be calculated as described above, the statistical distribution function of the overall cost for each microinsurer turns out to be too complex for an analytical solution under most realistic scenarios.[9] As explained earlier, the decision to sign a reinsurance treaty requires a lower reinsurance premium payment than the safety margin the microinsurer must maintain to ensure the same level of solvency that the reinsurance guarantees. But when the statistical distribution function of each microinsurer's business results are unknown or

TABLE 7.1 Reinsurance Results under Two Scenarios

Reinsurer's constraints	*Scenario 1 Reserves = Administrative costs Bankruptcy < 5 percent*		*Scenario 2 Reserves = Administrative costs + 0.5/microinsurer Bankruptcy < 5 percent*	
Number of microinsurers	36	100	36	100
Microinsurer's reinsurance threshold + reinsurance premium (₪)	5 + 1.69 = 6.69	5 + 1.52 = 6.52	5 + 1.19 = 6.19	5 +1.02 = 6.02
Failure rate without reinsurance (percent)	33	35	38	48
Failure rate with reinsurance (percent)	0	0	0	0

unsolvable, its business results cannot be estimated for a general solution over several periods and at different income levels, nor can the reinsurance premium be calculated. The premium calculation becomes even more difficult when the reinsurance pools fewer than 30 microinsurers with a heterogeneous risk profile (as is the case under most realistic scenarios), because the law of large numbers cannot be applied either. This is why it is necessary to resort to simulations (Monsef 1997) in an effort to determine the reinsurance premium. The results of the simulations under selected scenarios are described in the next section.

SIMULATION OF THE RELATIONSHIP BETWEEN MICROINSURERS AND REINSURER

Like analytical calculations, the simulation requires an estimate of the laws of distribution of cost-generating events[10] and their unit cost.[11] To recapitulate the methodology for analytical calculation of the effects of reinsurance, an example is provided as annex 7D, in which all microinsurers are assumed to be identical, and it is also assumed that their number is large (more than 30) to allow the application of the law of large numbers. The weakness of the example is that it does not allow drawing a decision rule for the general case, including the case where the microinsurers are not identical or are fewer than 30. If this is the reinsurer's reality, then the algebraic expression of the distribution of these random variables becomes too complex for an analytical solution. The alternative way to proceed then is to simulate the estimated business results of the applicant microinsurers.[12] To probe the reinsurance model through simulations, the Monte Carlo simulation method[13] has been applied.

One assumption of the simulations is that cost-generating events are independent of each other.

The results of the simulation provide information on:

- The failure rate for microinsurers that did not reinsure over several periods, but used the premium amount as additional income;
- The reinsurance premium payable by each microinsurer for each period;
- The advantages reinsurance provides microinsurers: distribution (mean, standard deviation, minimum, and maximum) of each microinsurer's expected surplus, protection against insolvency, and parameters influencing the microinsurer's utility;
- The distribution of the reinsurer's business results at the end of each period (mean, standard deviation [SD], minimum and maximum) and the reinsurer's survival probability; and
- The impact of external funding on reinsurance.

The simulations were run with computer software, elaborated in Delphi, programmed to respond to these specifications and called the *Toolkit* (described in appendix B).

SIMULATION RESULTS

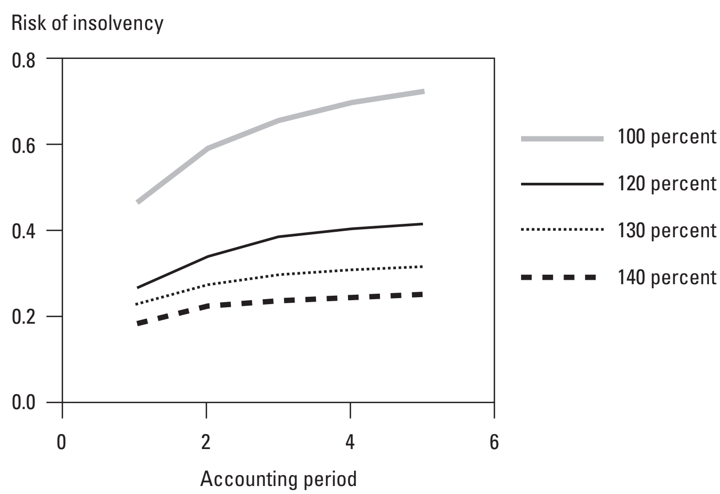
In the following section we present the answers obtained through simulations corresponding to the five bulleted points above, representing different aspects of the interactions between the microinsurers and the reinsurer.

Question 1: What is the failure rate of microinsurers that are not reinsured?

The microinsurer's survival is secured so long as it is solvent. In large schemes, there is a prevalent assumption that solvency is secured when income covers costs *(recovery rate),* which is usually estimated to equal the average cost over time. A similar logic has been applied to the microinsurer's solvency as a function of its income over five accounting periods. The question was explored first in relation to a concrete example of one microinsurer with 500 members, covering one risk, with probability of 1 percent (one event per 100 members per period), with average unit cost of 15 ₪. Hence, the average benefit cost for the microinsurer was 75 ₪ per period; the SD (obtained by simulating this microinsurer's business results in 2,500 replications) was 35.70 ₪ (47.6 percent of average cost).

In figure 7.2, four income levels were compared: 100 percent recovery rate,[14] 120 percent, 130 percent, and 140 percent (representing the recovery rate and, respectively 20, 30, and 40 percent above full recovery). The microinsurer is

FIGURE 7.2 Risk of Insolvency as a Function of Available Resources
(percent)



exposed to a high failure rate at every income level examined. Even when its revenue is 40 percent above recovery, the microinsurer can reach a failure rate of 19 percent at the end of one accounting period and 25 percent at the end of five periods. When the income is fixed at the more likely level of the recovery rate, the microinsurer's failure rate is 47 percent at the end of one accounting period and as high as 73 percent at the end of five periods.

This simulation provides two main insights. First, small microinsurers are vulnerable to insolvency, which cannot be remedied simply by increasing contribution levels. Second, this risk of failure worsens over time.

This figure does not offer any insight as to the reason for this vulnerability. However, seeing that microinsurers are usually small, could this vulnerability be linked to group size? This was explored in a second simulation, designed to see whether the same pattern applies to microinsurers of different sizes. Three microinsurers were compared: 200 members, 1,000 members, and 5,000 members; risk probability and unit cost were kept unchanged. The results of this simulation, shown in figure 7.3A, suggest that larger groups are less vulnerable to insolvency for a comparable income level. At an income level of 114 percent of recovery rate, the risk of failure was 22 percent, 44 percent, and 61 percent, respectively, for groups with 5,000, 1,000, and 200 members.

The three microinsurers exhibit almost identical results when income is expressed in terms of SD of the mean cost (figure 7.3B). As each microinsurer has a different simulated value of SD, expressing the microinsurer's resources in terms of multiples of its SD-enabled comparison. For this purpose, we define a coefficient Ω as a multiple of the SD of the total benefit cost. The income for all microinsurers was expressed as their recovery rate (100 percent) plus Ω multiplied by SD. The risk of failure under this representation, at $\Omega = 0.5$, was 45 percent, 44 percent, and 47 percent, respectively, for the microinsurer with 5,000, 1,000, and 200 members. This result suggests that the difference in failure rates between these microinsurers (figure 7.3A) is solely the result of the difference in their variance in benefit cost, which is very sensitive to group size (Dror, chapter 5, this volume).

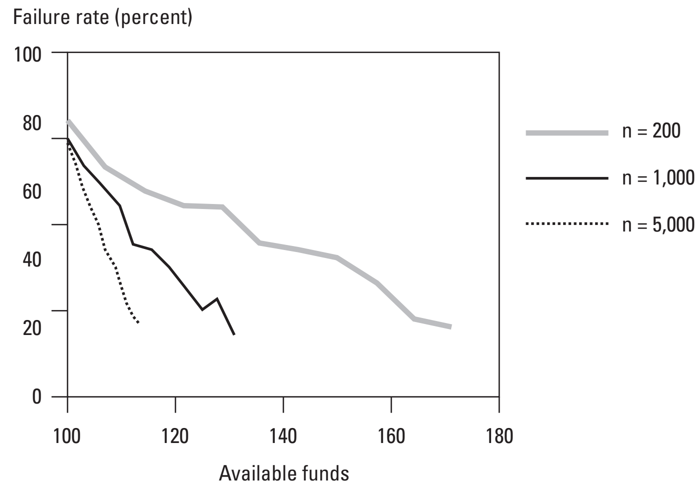
In conclusion, the reply to the question "What is the failure rate of microinsurers that are not reinsured?" is that the failure rate is too high to ignore. The high risk of failure applies in all cases but will be accentuated by small group size, higher risk profile, and lower income.

This leads to a search for a sustainable and affordable solution. If reinsurance might be such a solution, its affordability needs to be assessed, which is the topic of the next section.

Question 2: What level of premium should be set for the microinsurer's reinsurance?

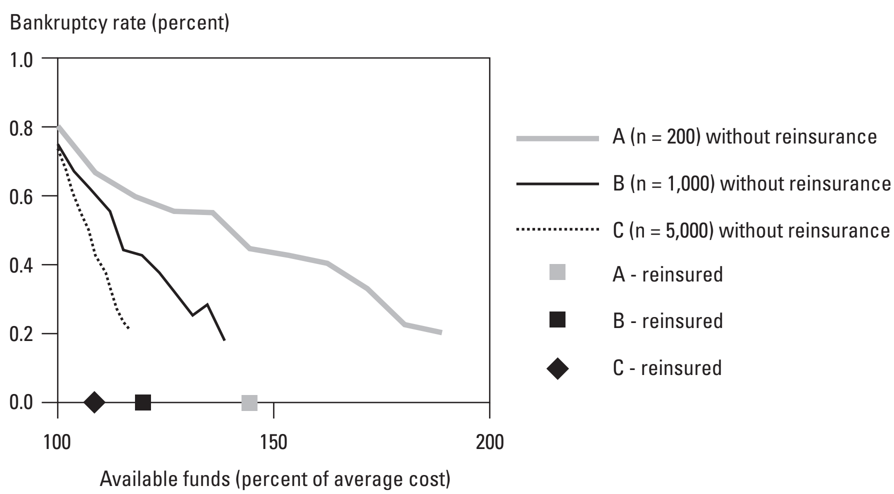
Under the reinsurance contract, the reinsurer agrees to pay benefit costs exceeding the expected average (75 ₪), whereas the microinsurer agrees to pay benefit

FIGURE 7.3A Failure Rate of Microinsurance Units



Note: Failure expressed in terms of recovery rate.

FIGURE 7.3B Bankruptcy Rate of Microinsurance Units with and without Reinsurance



Note: Five accounting periods.

costs up to that level, plus a reinsurance premium. This contract guarantees that the microinsurer would never become insolvent as a result of above-average costs. The premium is set at a level that guarantees the reinsurer's solvency at 95 percent. The objective of the simulation is to explore the lowest value that can satisfy this requirement.

As in the discussion of the previous question, we revert initially to one microinsurer with 500 members, with the same risk profile used above (p = 1 percent, unit cost = 15 ₪, SD = 35.70 ₪). And as the reinsurer operates a pool of several microinsurers, six pooling options were simulated (5, 20, 30, 40, 50, or 75 identical microinsurers in the pool).

As the risk of each microinsurer is proportional to the variance of its benefit cost, the reinsurance premium is also a function of the variance, and expressed as Ω multiplied by the SD. The range of acceptable solutions has been calculated for values of Ω between [0,1]. The results are illustrated in figure 7.4.

As can be seen in the figure, the size of the pool makes a moderate difference: when only 5 microinsurers are pooled, the premium to ensure the reinsurer a 95 percent survival rate is at least 0.6*SD (28.6 percent of the average cost). When the pool includes 20 microinsurers, the premium drops to 0.5*SD (about 23.8 percent of the average cost of the stereotypic microinsurer described above[15]). When the pool increases further, the premium does not decrease below Ω = 0.5. The simulation was performed for many scenarios, with different microinsurer sizes and risk profiles. In every case, the lowest premium (at an optimal pool size) was consistently 0.5*SD.

As the variance determines the reinsurance premium, and the size of the microinsurer's membership has an impact on its SD, the premium would be expected to cost a microinsurer with a larger membership less than a microinsurer with a smaller membership. This conjecture was verified in another simulation, the results of which are illustrated in figure 7.5. As can be seen in the figure, when the microinsurer has only 200 members, it must pay 37.6 percent of its contribution income as reinsurance premium; but when the number of members increases

FIGURE 7.4 Levels of Reinsurance Premium Securing 95 Percent Survival of Reinsurer

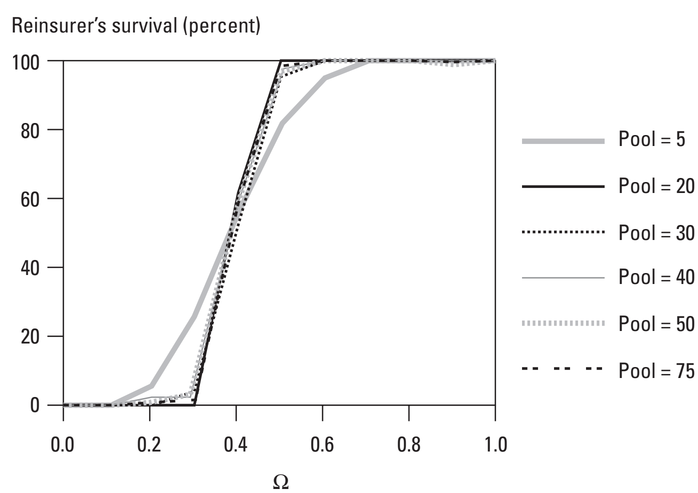
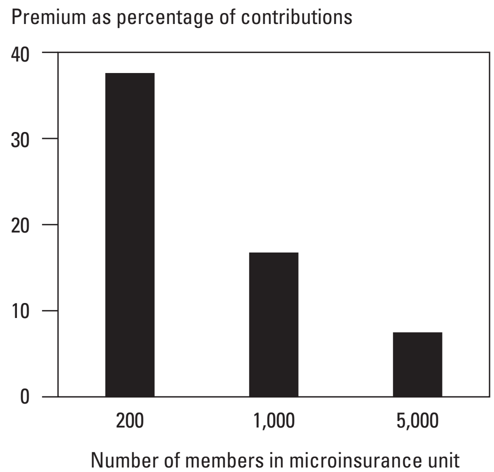


FIGURE 7.5 Effect of Group Size on Premium



to 1,000, the premium represents 16.8 percent of contribution income, and it drops to only 7.5 percent when the group size is 5,000, for an identical risk profile in every instance.

So far we have dealt with a homogeneous pool of microinsurers, all sharing the same group size and risk profile. In reality, we may expect heterogeneity of both parameters. The impact of such heterogeneity will be explored using a pool of five microinsurers with different profiles as an example (table 7.2). The same distribution laws used before are used here, too: Poisson law for occurrence distribution and Chi-square for unit cost distribution.

Table 7.2 provides the mean cost of benefits, but not the indispensable information about variance of this value. These values, obtained by applying the general mathematical expression (annex 7C) are provided in table 7.3.

Table 7.3 provides the SD of the mean benefit cost for the five microinsurers, in nominal terms and as a function of the mean. Contrary to the homogeneous microinsurers, these microinsurers differ greatly from each other in terms of the value of their mean (from 2.5 to 36) and in their SD (48 percent to 213 percent of mean). The premium that would be required to satisfy the reinsurer's solvency rate of 95 percent has been simulated for this group. The striking result is that for this pool, the minimum premium required is 0.9*SD (compared with the 0.6*SD required for a pool of five microinsurers with a homogeneous risk profile). Note that the total number of individuals covered by the two pools is similar (2,300 versus 2,500), and the claim load is also almost identical (25 versus 25.1). The results, shown in figure 7.6, should be compared with those in figure 7.4.

In conclusion, the reinsurance contract, as defined above, assumes that the premium is proportional to the risk ceded to the reinsurer. It has been shown that this risk is proportional to the SD of each microinsurer's total benefit cost. As also is

TABLE 7.2 Microinsurers' Characteristics in the Simulation

		Cost-generating event 1		*Cost-generating event 2*		
	Number of members	*Mean occurrence (cases per period)*[a]	*Mean unit cost (₪)*	*Mean occurrence (cases per period)*[a]	*Mean unit cost (₪)*	*Reinsurance threshold = Mean benefits (₪)*
Microinsurer-1	100	1	1	0.1	15	2.5
Microinsurer-2	1,000	8	2	2.0	10	36
Microinsurer-3	150	2	3	0.5	12	12
Microinsurer-4	300	4	4	0.5	30	31
Microinsurer-5	750	6	3	1.0	10	28

a. This represents the number of events for the microinsurer, which also depends on the number of members, to express mean occurrence per member. This number would be divided by the number of members in the microinsurance unit.
Note: The numbers are the mean of three runs. The data used here are theoretical, and it is assumed that the two cost-generating events, their unit cost, and the incidence are independent. This assumption of independence between events is a technical one, allowing random numbers to be generated in a simple way. Introducing correlation between cost-generating events, although possible when necessary, would vastly complicate the process of random number generation.

TABLE 7.3 Distribution of the Benefit Cost

	SD of benefits (₪)	*SD (percent of mean)*
Microinsurer-1	5.33	213
Microinsurer-2	17.44	48
Microinsurer-3	10.68	89
Microinsurer-4	24.00	77
Microinsurer-5	14.49	52

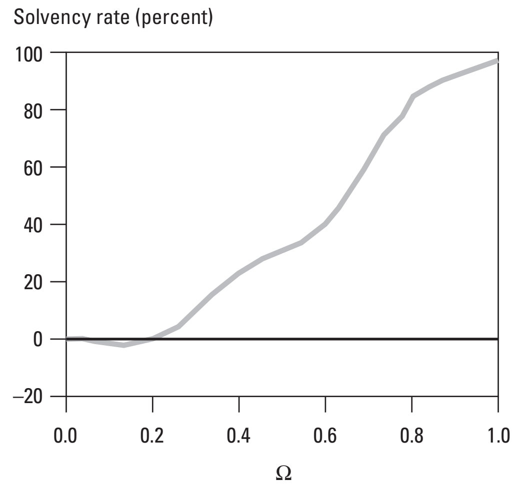
shown, the SD decreases (per person) when group size increases, all else remaining unchanged. Hence, the reinsurance premium decreases in terms of its share of total expenses as membership increases. An increase in the number of microinsurers pooled through reinsurance also reduces the premium, as the risk is spread over a larger population. However, even when the pool is large enough, the premium does not decrease below half the SD of each microinsurer's total cost. Finally, the vulnerability of small pools increases dramatically when the risk profile is heterogeneous.

Now that the cost of the premium has been identified, the question is whether it is worth paying. Stated differently, the decision to reinsure or not will depend on two considerations: on the premium and on the return microinsurers can expect for this payment.

Question 3: What Do Microinsurers Get Out of Reinsurance?

Reinsurance gives microinsurers both protection against insolvency and some discretionary budget.

FIGURE 7.6 Reinsurance Premiums for a Heterogeneous Pool of Microinsurers



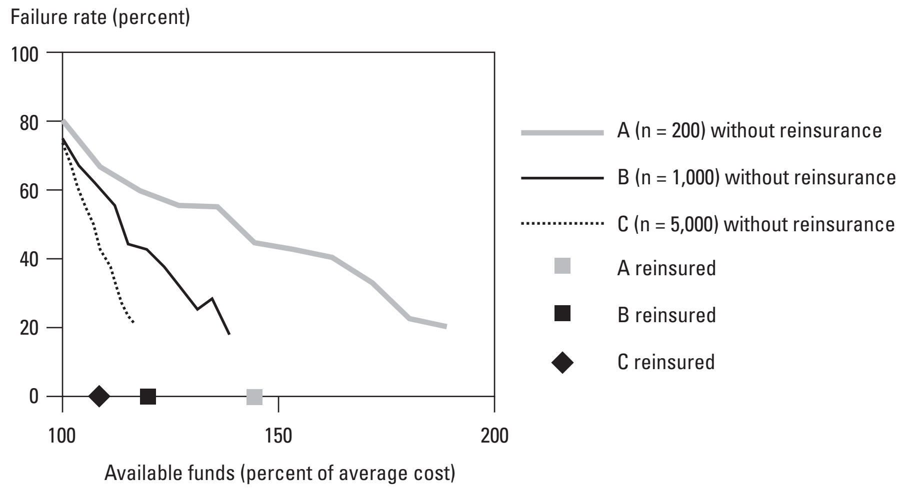
Protection against insolvency. Reinsurance reduces the microinsurer's risk of insolvency. This is its fundamental advantage. Figures 7.2, 7.3A, and 7.3B show that, without reinsurance, the risk of failure is very high, even with enough resources to secure a full recovery rate.

As the reinsurance contract guarantees that the reinsurer pays all costs above the reinsurance threshold, the microinsurer's risk of failure is eliminated. The microinsurer has to decide whether the cost of the premium compares favorably to the safety margin it must preserve (which is proportional to the variance of its benefits). In the following set of simulations, we compared the use of the premium amount (assuming an optimal pool size) to the use of an identical amount as a safety margin. The results are shown in figure 7.7.

The figure shows the same groups depicted in figure 7.3. As can be seen now, for group n = 1,000, the premium was 16.8 percent of the recovery rate. Using this amount as a safety margin would reduce the risk of failure from 73 percent to 44 percent at the end of five periods. Using the same amount to pay the reinsurance premium would reduce the failure rate from 73 percent to 0 percent from the first period. The same utility, observed for all three microinsurers, is related to setting the premium at 0.5*SD (as shown in figure 7.3B), even though its nominal level differs because of each microinsurer's particular features. Therefore, reinsurance presents a clear advantage for all microinsurers, regardless of their specific features.

Discretionary budget. As mentioned earlier, in good years, microinsurers would run a surplus because their actual costs would be lower than the mean. Although the probability of good years is unaffected by reinsurance, this financial safety releases

FIGURE 7.7 Comparison of the Premium to the Safety Margin



microinsurers from the need to maintain contingency reserves, thus allowing them to use these surpluses as discretionary budgets without taking any additional risk of failure. It seems fair to assume that the larger this financial resource, the more attractive microinsurers would find reinsurance.

The size of the discretionary budget has thus been simulated for each microinsurer and period. Bearing in mind that the premium level is influenced by the microinsurer's membership size, this simulation also looked at three levels, with n = 200, n = 1,000 and n = 5,000. The discretionary budget was shown as a proportion of the premium paid. The results of the simulation are shown in figure 7.8.

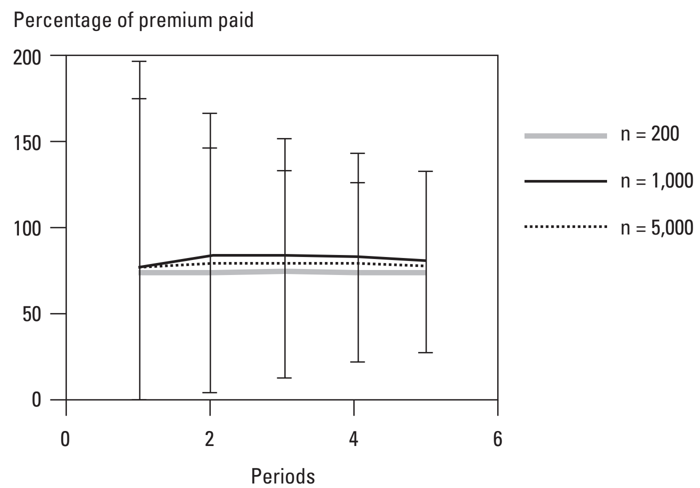
Quite unexpectedly, the discretionary budget seems to represent about 80 percent of the premium in every case, regardless of group size and of the period covered. As can be seen in the figure, the likelihood of accumulating a discretionary budget increases over time, because its SD, although high initially, drops over time. These simulations were run on the assumption that the pool size was optimal (premium = half the SD). Incidentally, this simulation was repeated with other variables (risk levels, number of risks in the benefit package), and in every case, the accumulation was, on average, around 80 percent of the premium paid.

The practical implications of this finding have been explored further by looking at four microinsurers with different characteristics (table 7.4).

In terms of perceived utility, microinsurers look at two aspects: first, the premium they need to pay to avoid failure and secure full solvency; second, the amount of discretionary budget they can obtain.

Microinsurers A and B share the same risk profile, but the difference in their membership is tenfold. This membership differential accounts for their respective

FIGURE 7.8 Discretionary Budget



SD values of 47.6 percent and 15.1 percent of mean cost, respectively, which also explains the more-than-threefold difference in the premium. On the other hand, the discretionary budget also dropped, from 0.14 ₪ to 0.05 ₪ per member.

Microinsurers B and C differ in the number of members and in risk probability but have the same claim load. Both pay the same premium in terms of percentage of mean cost, but C's discretionary budget is 10 times higher per member than that of B. C's perceived utility would thus be higher than B's.

TABLE 7.4 Premium and Discretionary Budget for Different Microinsurers

Microinsurer's characteristics	*A*	*B*	*C*	*D*
Members	500	5,000	500	500
Number of benefits	1	1	1	3
Risk probability (percent)	1	1	10	P1=P2=P3=1
Mean claim load	5	50	50	15
Unit cost	15	15	15	15
Premium and discretionary budget data				
Mean total cost of benefits	75	750	750	225
Standard deviation	35.70	112.91	112.91	61.84
Standard deviation/mean cost (percent)	47.6	15.1	15.1	27.5
Premium (percent of mean cost)	23.8	7.5	7.5	13.9
Discretionary budget per member after five periods	0.14	0.05	0.45	0.25

Microinsurers A and C have the same membership but a different risk profile. Microinsurer A reinsures a rare event, whereas C reinsures a more frequent risk. Microinsurer A therefore pays a higher share of its expenditure as premium (47.6 percent versus 15.1 percent), but as this premium is lower than C's in nominal terms, A can expect a lower discretionary budget.[16]

Finally, a comparison of microinsurers A and D reveals another interesting aspect. Both microinsurers have the same group size, but A has one benefit whereas D has three. This risk diversification causes a threefold increase in claim load, a decrease in D's relative premium, and an increase in the discretionary budget. Once again, we conclude that D has a higher perceived utility than A from reinsurance.

This analysis suggests that the higher the microinsurer's claim load, the lower is the share of the premium *relative* to the microinsurer's total expenditure. Also, the higher the claim load, the higher is the discretionary budget because it is linked to the *nominal* value of the premium, which is higher. Furthermore, it seems that subscribing to reinsurance would be an optimal policy choice for microinsurers that offer a package with many benefits, including some that are not rare.

If the reinsurer is not solvent at all times, however, reinsurance would be impossible. This will be explored next.

Question 4: What is the reinsurer's balance and risk of insolvency?

The premium calculation has been based on the assumption that, in the long run, the reinsurer's risk of insolvency should not exceed 5 percent. For this purpose, it is not enough for the reinsurer's mean balance to be positive; the worst-case scenario has to be positive as well. If the reinsurer's business results in any one period were negative, reinsurance could, however, still work as long as enough resources were available to cover any operational deficit. In accounting terms, this translates into a requirement that the reinsurer's business results should be measured on the basis of accrued accounts, with surpluses and deficits transferred across accounting periods.

The reinsurer's balance has been simulated on the accrual basis, with the same specifications as those used for figure 7.4, but with only two pool sizes: 5 and 20 microinsurers. The new simulations show the reinsurer's business results during the first five periods. Figure 7.9 illustrates the balance in relation to the ceded risk. The ceded risk has been defined here as one-and-a-half times the sum of the standard deviation of the total cost of affiliated microinsurers. Figure 7.9A depicts the situation for a pool size of five microinsurers, and at two premium levels: 0.5*SD and 0.6*SD. Although the mean simulated balance is positive at both premium levels, the lower end of the variance of the results (that is, the worst-case scenario) at 0.5*SD clearly exposes the reinsurer to a negative balance in all years. The only business result that overcomes this limitation is achieved, in Period 5, when the premium is set at 0.6*SD. Figure 7.9B shows the reinsurer's balance at a pool size of 20 microinsurers. Here, the mean is identical to that of the smaller

FIGURE 7.9A Reinsurer's Balance, Pool of Five Microinsurers

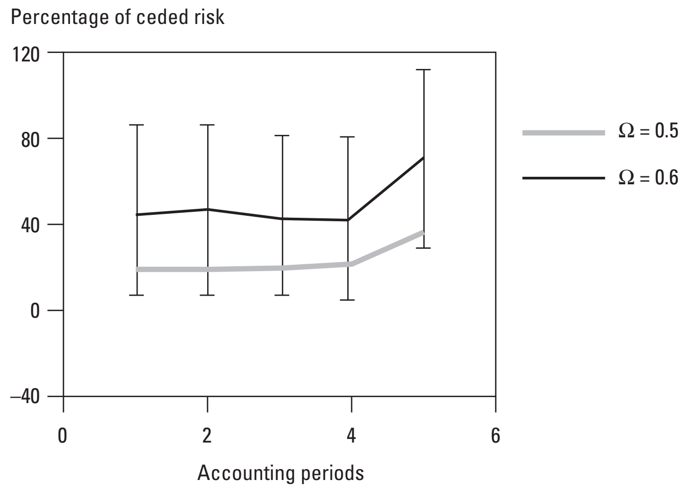
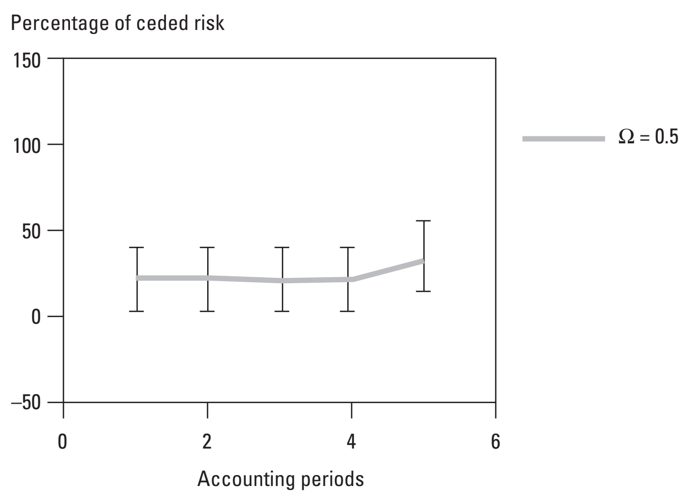


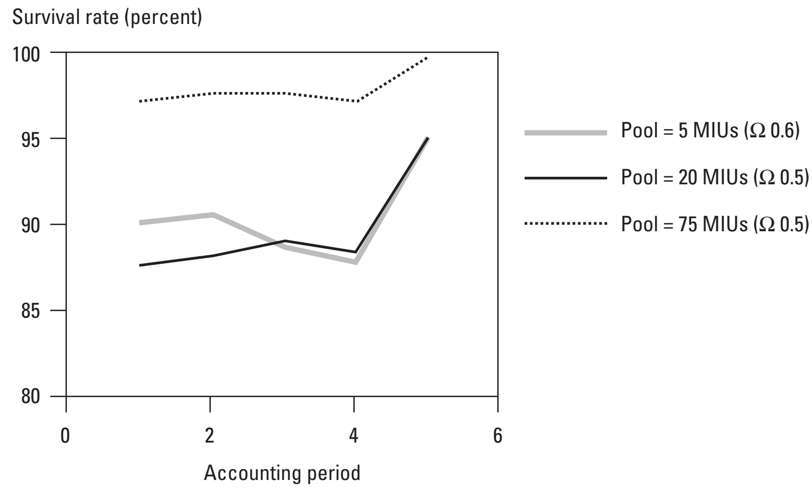
FIGURE 7.9B Reinsurer's Balance, Pool of 20 Microinsurers



pool, but because the larger pool size greatly reduces the variance, the reinsurer's viability can be secured even with a premium of 0.5*SD. This result should be treated with some caution, however, as it may be different when the risk profile of the participating microinsurers is heterogeneous.

The variance discussed above implies that the reinsurer's solvency rate may be below the required level of 95 percent. The same simulation results were therefore used to derive the solvency rate. The results are presented in figure 7.10.

FIGURE 7.10 Reinsurer's Solvency



Here, the solvency rate does not reach the required 95 percent before the end of Period 5, when the pool is composed of 5 or 20 microinsurers. For the sake of comparison, the pool of 75 microinsurers is also shown. This large pool can secure the required solvency rate from the first period with a premium of 0.5*SD (but not with a lower premium—as already shown in figure 7.4).

In conclusion, the condition of 95 percent solvency is not met during the first four periods when the pool is no larger than 20 microinsurers and the lowest possible premium is set (discussed under Question 2). When the pool includes 75 microinsurers, this problem is eliminated, and the reinsurer can reach solvency of 95 percent or more from the first period.

As a pool that large at start-up of reinsurance seems unlikely, an alternative solution to secure the reinsurer's required solvency rate might be to begin operations with sufficient funds to provide the necessary financing for worst-case scenarios during the first four periods. The impact of external funding on the reinsurance operation is discussed in the next section.

Question 5: What impact does external funding have on reinsurance?

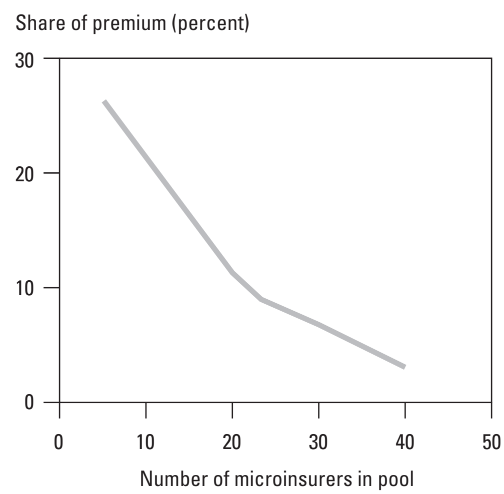
As we have seen, small microinsurers joining small reinsurance pools are exposed to a dual vulnerability. First, their variance is likely to be high, which translates into higher premiums, because these are calculated on the basis of variance. Second, small pools also cause an increase in the premium to secure the reinsurer's solvency at 95 percent from start-up. Since reinsurance for community-based schemes is likely to start with a small pool, quantifying the small-size surcharge and finding ways to release microinsurers from it would be desirable. Such an approach would be in line with the purpose of reinsurance as a mechanism offering financial sustainability to microinsurers at an affordable cost.

The question thus is: What size subsidy will both allow each microinsurer to pay only the minimum premium (0.5 of its SD) and also secure the reinsurer's solvency at 95 percent from the first period on? This quantity was obtained by comparing the full premium needed to the reduced premium over the first five periods. The results are shown in figure 7.11. As can be seen, the subsidy would be 26 percent of the premium for the stereotypic microinsurer[17] when the pool includes only 5 microinsurers. The subsidy would drop to 3 percent when the pool increases to 40 microinsurers. It stands to reason that this level of subsidy would be different, probably higher, when the pool is composed of microinsurers with a heterogeneous risk profile. Also, after five years, the subsidy would no longer be necessary because the reinsurer would be financially self-sufficient at the same premium for any pool larger than 20 microinsurers.

This insight points to the important impact of external resources on the way to achieve equity among microinsurers buying reinsurance. Once such resources are available, the reinsurer can negotiate the same premium level with each microinsurer regardless of the pool it affiliates to.

Besides the premium subsidy, the reinsurer needs to secure resources to pay for risk-management services (discussed in Feeley, Gasparro, and Snowden chapter 22, this volume).

FIGURE 7.11 Subsidy Needed to Limit the Premium to $\Omega = 0.5$



CONCLUSIONS

This model offers a way to quantify microinsurers' vulnerability and to examine the effectiveness of reinsurance as a remedy to it. The discussion has been limited to considerations that can be predicted by the application of statistical laws, and the focus has been on the effect of fluctuations in the microinsurers' and the reinsurer's total benefit expenditures.

The model assumes that microinsurers can fund their mean cost of benefit expenditure.[18] This assumption allows comparison of microinsurers and large health schemes, where it is often assumed that revenues should equal cost recovery. The simulations have shown that, even if this condition is satisfied, only 20 percent of the microinsurers will avoid insolvency within a time frame of five accounting periods. This longer-term view of community-based schemes is relatively rare in the literature, where operations are more often described for short or unspecified time frames, when accidental clustering of cost-generating events may cause insolvency. It has been shown that claim load is the critical predictor of the microinsurer's financial situation. When the claim load is relatively low or when the variance in total cost is relatively large, microinsurers cannot stabilize their financing autonomously, and reinsurance can provide financial stabilization.

Variance in total cost can stem from a small claim load or from a great variation in unit cost. A small claim load is likely to occur either when the group is very small or when the event is very rare. The model described here is suited to all these circumstances.

The reinsurance model can be applied when the SD of each affiliated microinsurer's total benefit cost is known. The reinsurer's success is highly sensitive to the accuracy of this SD; a 20 percent error in SD value can signify the difference between the reinsurer's long-term solvency or bankruptcy. The SD can be calculated only when the risk probability is known. In reality, the estimate of risk is often unreliable, and methods for improving it (discussed in Auray and Fonteneau, chapter 8), are likely to be introduced only when support for risk-management techniques is available.

Even when risk probability is known, the reinsurer is still affected by affiliated microinsurers' pool size and the heterogeneity of risk profiles. The larger the pool, the better the reinsurer can spread risk and thus reduce the variance of its business outcome. When the pool is small, the effect of heterogeneous risk profiles requires a higher premium for stabilization.

Therefore, reinsurance may be very vulnerable in its initial years of operation, even if the pool is large. To remain financially stable during these years, the reinsurer must secure sufficient reserves to cover unpredictable error in risk estimates and the possibility that developing an optimal pool size might take several years.

In any event, reinsurance premiums cannot be set lower than half the SD of each microinsurer's total cost, and several circumstances require higher premiums.

If every microinsurer paid the same minimal rate, subsidies would have to cover the difference in the reinsurer's income for the first four years.

Reinsurance is useful for two main reasons: It helps improve the microinsurer's solvency and gives the microinsurer access to resources from accumulated surpluses that can be spent instead of held as reserves. While both components are important and interrelated, enhanced access to resources would likely appeal more to the reinsurer's potential clients, since discretionary budgets seem more tangible than protection against a risk that is hard to visualize. On the other hand, policymakers and donors may be more interested in securing microinsurers' long-term viability, and a subsidy to support this development may prove more attractive to microinsurers than ongoing financial support for their continued operations.

Some of the proposed model's limitations should also be recalled. First, the model does not deal with catastrophic risk.[19] Second, some statistical assumptions about real-life situations may be weak, for instance, the assumption that events are independent or that all unit costs are distributed according to a single law. These assumptions, necessary at the conceptualization phase, should be verified during piloting for each microinsurer. Third, the model assumes a distinction between insurable and uninsurable events and deals only with the former. This taxonomy (discussed in Vaté and Dror, chapter 6, this volume) does not always offer a clear-cut distinction between the two types of event. Finally, the administrative costs of operating reinsurance have been ignored in the simulations described here. In the long run, however, these costs would have to be covered by premium income or through other resources and would have to be affordable.

ANNEX 7A A MATHEMATICAL MODEL

Let us consider n microinsurers and T periods. We observe that:
$X(i, t, .)$ represents the aggregate benefits that microinsurance unit (microinsurer) i must pay at the end of period t. The dot indicates that at the beginning of the period this amount is unknown. We assume that $X(i, t, .)$ is a random variable for which the distribution function $F(i, t, x)$ is known. $\overline{X}(i,t)$ designates the mean of $X(i, t,.)$.

$m[\overline{X}(i,t)]$ is the amount set up as reserves by the microinsurer at the beginning of period t to pay its aggregate benefits for period t. This sum shall be expressed in relation to $\overline{X}(i,t)$.

$h[\overline{X}(i,t)]$ is the sum the microinsurer must pay before it can claim from the reinsurance. This amount has been called the *reinsurance threshold*.

We assume that $m[\overline{X}(i,t)] \geq h[\overline{X}(i,t)]$.

$\Delta(i, t)$ is the premium paid by microinsurer i to the reinsurer for period t.
Microinsurers may reaffiliate for t periods, $t = 1, \ldots, T$.

Case A1—Microinsurers n sign a reinsurance contract for one period.

If microinsurer i did not sign a reinsurance contract, its probability of survival α_{i1} is equal to:

$$\alpha_{i1} = P[m[\overline{X(i,1)}] - X(i, 1, .) \geq 0] = P[X(i, 1, .) \leq [m[\overline{X(i,1)}]].$$

Let us calculate the probability of survival β_{1i} in the case where the microinsurer decides to sign a reinsurance contract. For this, we define random variable Z(i, t, .) as follows:

$$Z(i, 1, .) = \begin{cases} m[\overline{X(i,t)}] - X(i, t, .) - \Delta(i, t) & \text{if} \quad X(i, t, .) \leq h[\overline{X(i, t)}] \\ m[\overline{X(i,t)}] - h[\overline{X(i,t)}] - \Delta(i, t) & \text{if} \quad X(i, t, .) > h[\overline{X(i, t)}] \end{cases}$$

$$\beta_{1i} = P[Z(i, 1, .) \geq 0]$$

And:

$$P[X(i,1,.) \leq \text{Min}\{[m[\overline{X(i,1)}] - \Delta(i,1), h[\overline{X(i,1)}]\}] + P[\{m[\overline{X(i,1)}] - h[\overline{X(i,1)}] - \Delta(i,1) \geq 0\} \cap \{X(i,1,.) > h[\overline{X(i,1)}]\}]$$

Yet if:

$$\Delta(i,1) > m[\overline{X(i,1)}] - h[\overline{X(i,1)}] \text{ then:}$$

$$P[\{m[\overline{X(i,1)}] - h[\overline{X(i,1)}] - \Delta(i,1)0\} \cap \{X(i,1,.) > h[\overline{X(i,1)}]\}] = 0$$

if

$$\Delta(i,1) \leq m[\overline{X(i,1)}] - h[\overline{X(i,1)}] \text{ then:}$$

$$P[\{m[\overline{X(i,1)}] - h[\overline{X(i,1)}] - \Delta(i,1)0\} \cap \{X(i,1,.) > h[\overline{X(i,1)}]\}] = P[X(i,1,.) > h[\overline{X(i,1)}]]$$

Where:

If $\Delta(i,1) > m[\overline{X(i,1)}] - h[\overline{X(i,1)}]$ then $\beta_{1i} = P[X(i,1,.)\ m[\overline{X(i,1)}]] - \Delta(i,1)]$

If $\Delta(i,1) \leq m[\overline{X(i,1)}] - h[\overline{X(i,1)}]$ then $\beta_{1i} = 1$

Thus:

If $\Delta(i,1) > m[\overline{X(i,1)}] - h[\overline{X(i,1)}]$ and $\alpha_{i1} \geq \beta_{1i}$, then microinsurer i has no interest in reinsuring itself.

If $\Delta(i,1) \leq m[\overline{X(i,1)}] - h[\overline{X(i,1)}]$ and $\beta_{1i} = 1$, then microinsurer i has every interest in reinsuring itself as this will guarantee its survival.

Let us now look at the problem of the reinsurer.

Benefits paid to microinsurer i at the end of period t is a random variable that we designate by W(i, t, .) and which is equal to:

$$\mathbf{W(i, t, .)} = \begin{cases} 0 & \text{if } \quad X(i, t, .) \leq h[\overline{X(i, t)}] \\ X(i, t, .) - h[\overline{X\ (i,t)}] & \text{if } \quad X(i, t, .) > h[\overline{X(i, t)}] \end{cases}$$

It can be proven that the distribution function of variable W(i, t, .), noted as G(i, t, w) is equal to:

$$G(i, t, w) = \begin{cases} 0 & \text{if } \quad w < 0 \\ F(i, t, h[\overline{X\ (i,t)}] & \text{if } \quad w = 0 \\ F(i, t, w + h[\overline{X(i, t)}] & \text{if } \quad w > 0 \end{cases}$$

Consequently, if $\overline{W\ (i,t)}$ is the mean of W(i, t, .) and σ^2_{Wit} its variance, then:

$$\overline{W\ (i,t)} = \int_{-\infty}^{+\infty} wdG(i, t, .) = \int_{0}^{+\infty} wdF(i, t, .) \text{ and } \sigma 2Wit = \int_{-\infty}^{+\infty} w^2dG(i, t, .) - \overline{W\ (i,t)}^{\,2}$$

Let B_0 designate the funds the reinsurer has on hand at the onset of its activity, and s(n, t) its administrative costs for the period t when it reinsures n microinsurance units.

We note that A(n, t) equals $B_0 - \sum_{j=1}^{t} s(n, j)$,

The insurer's probability of survival at the end of the first period, γ_1 is equal to:

$$\gamma_1 = P[A(n, 1) + \sum_{i=1}^{n} \Delta(i, 1) - \sum_{i=1}^{n} W(i, 1, .) \geq 0\,] = P[\,\sum_{i=1}^{n} W(i, 1, .) \leq A(n, 1) + \sum_{i=1}^{n} \Delta(i, 1)]$$

The analytical expression of the convolution product law for random variables W(i, 1, .) is obtainable only in specific cases.

Example

Here we assume, regardless of microinsurer i, that X(i, t, .) = X(t, .) and random variables X(i, t, .) are independent pairwise. We know (Bass 1974) that if n is greater than or equal to 30, a normal law containing mean $\overline{W\ (i,t)}$ and variance σ^2_{Wit}/n can approximate the law of random variable Z_{nt} defined by $1/n \sum_{i=1}^{n} W(i, t, .)$

In this case:

$$\gamma_1 = P[Z_{n1} \leq 1/n\ [A(n, 1) + \sum_{i=1}^{n} \Delta(i, 1,)]]$$

As the distribution of financial risk is the same for each microinsurer, we can reasonably put forth the hypothesis that they will all obtain the same contract in terms of premiums and reinsurance thresholds. Thus, regardless of whether i = 1, …, n, we have $\Delta(i,1) = \Delta(1)$ and $h[\overline{X(i,1)}] = h[\overline{X(1)}]$, whereby:

$$\gamma_1 = P[Z_{n1} \le \frac{A(n, 1)}{n} + \Delta(1)] = P[\mathbf{Z}_{n1} \le [\frac{A(n, 1)}{n} + \Delta(1) - \overline{W(1)}]\sqrt{n}/\sigma_{W1}]]$$

where $\mathbf{Z}_{n1}$ is the random standard variable associated to Z_{n1} and $\overline{W(1)} = \overline{W(i,t)}$, $\sigma_{W1} = \sigma^2_{Wi1}$ regardless if i = 1, …, n.

If the reinsurer is willing to accept a bankruptcy risk of $0.05 = 1 - \gamma_1$ then we have:

$$[\frac{A(n, 1)}{n} + \Delta(1) - \overline{W(1)}]\sqrt{n}/\sigma_{W1} = 1.65$$

Where:

$$\Delta(1) = \overline{W(1)} - \frac{A(n, 1)}{n} + 1.65\sigma_{W1}/\sqrt{n}$$

and, as the microinsurer is interested in reinsurance only when $\Delta(1) < m[\overline{X(1)}] - h[\overline{X(1)}]$, this reinsurance model can be considered only if:

$$0 < \overline{W(1)} - \frac{A(n, 1)}{n} + 1.65\sigma_{W1}/\sqrt{n} < m[\overline{X(1)}] - h[\overline{X(1)}]$$

Case A2—Microinsurers n sign a reinsurance contract for T periods, T > 1.

Let us calculate the survival probability β_{Ti} for microinsurer i in the case where it signs a reinsurance contract for T periods.

$$\beta_{Ti} = P[Z(i, 1, .) \ge 0, Z(i, 2, .) \ge 0, \ldots, Z(i, T, .) \ge 0] = \prod_{t=1}^{t=T} P[Z(i, t, .) \ge 0]$$

Consequently, regardless if t = 1, …, T, $\Delta(i,t) \le m[\overline{X(i,t)}] - h[\overline{X(i,t)}]$ the survival of the microinsurer i is certain throughout the entire period T.

To calculate the reinsurer's survival probability, we will suppose that the reinsurance contract binds each microinsurer to the reinsurer for T periods; that the amount of the premium for each period was determined when the contract was signed: and, to simplify matters, that the reinsurer's survival is calculated only at the end of period T. This last hypothesis implies, for example, that, in case of losses over one or more periods, the reinsurer can obtain interest-free financing of the deficit, repayable from profits in the future.

Within the context of these hypotheses, the reinsurer's survival probability at the end of period T is equal to:

$$\gamma_T = P[A(n,T) + \sum_{t=1}^{t=T}\sum_{i=1}^{n}\Delta(i, t) - \sum_{t=1}^{t=T}\sum_{i=1}^{i=n}W(i, t, .) \ge 0] = P[\sum_{t=1}^{t=T}\sum_{i=1}^{i=n}W(i, t, .) \le A(n,T) + \sum_{t=1}^{t=T}\sum_{i=1}^{i=n}\Delta(i, t)]$$

Example

If we look again at the example above, where we suppose that microinsurers n are similar, even if i = 1, …, n, X(i, t, .) = X(t, .), and if we suppose that time frame T is short enough for the microinsurers' benefit distributions to remain constant, that is X(t, .) = X(.), then:

W(i, t, .) = W(.) for all i = 1, …, n and all t = 1, …, T

If product nT is greater than 30, we know that random variable Z_{nT}, defined as $\frac{1}{nT}\sum_{t=1}^{t=T}\sum_{i=1}^{n} W(i, t, .)$, can be approximated with a normal law containing mean and variance σ^2_W/nT, where $\overline{W}$ and σ^2_W are, respectively, the mean and variance of W(.).

We then have:

$$\gamma_T = P[Z_{nT} \leq \frac{1}{nT}[A(n, T) + \sum_{t=1}^{t=T}\sum_{i=1}^{i=n} \Delta(i, t)]]$$

and if Δ(i, t) = Δ for all i = 1, …, n, t = 1, …, T, which seems reasonable in this case as the microinsurers are similar, we obtain, with a 95 percent survival probability for the reinsurer:

$$[\frac{A(n, T)}{nT} + \Delta - \overline{W}]\sqrt{nT}/\sigma_W = 1.65$$

Where:

$$\Delta = \overline{W} - \frac{A(n, T)}{nT} + 1.65\sigma_W/\sqrt{nT}$$

and the condition that guarantees the interest of the reinsurance process:

$$0 \leq \overline{W} - \frac{A(n, T)}{nT} + 1.65\sigma_W / \leq m[\overline{X}] - h[\overline{X}]$$

ANNEX 7B CALCULATING THE REINSURANCE PREMIUM

In the following simple example, the reinsurance premium can be calculated analytically.[20]

Let us consider a case where more than 30 identical microinsurers sign identical reinsurance contracts for one time period. The law of large numbers allows us to state the following: the reinsurer operates under the condition that its risk of bankruptcy will not exceed 5 percent. The reinsurance premium Δ payable by each microinsurer must satisfy the following equation:

$$\Delta = \overline{W(1)} - \frac{A(n, 1)}{n} + 1.65\sigma_{W1}/\sqrt{n}$$

Where:

$\overline{W(1)}$ is the mean of a random variable W(1) equal to benefits paid by the reinsurer during Period 1,

σ_{W1} is the SD of the random variable W(1),

n is the number of microinsurers pooled by the reinsurance contract,

A(n,1) is the reinsurer's initial capital, minus administrative costs for the first period.

The equation indicates that, when the number of reinsured microinsurance units increases and the reinsurer's initial capital increases more slowly, the reinsurance premium will lean toward $\overline{W(1)}$. In this case, reinsurance will be attractive for microinsurers only when the premium $\overline{W(1)}$ is lower than the safety margin (that is, proportional to the microinsurer's own benefit expenditure variance) and when the reinsurance threshold is equal to the mean cost of the microinsurer's benefits.

The next example illustrates a case where the distribution probability of each microinsurer's business results is available. A uniform distribution of benefits payable by the microinsurer to its members is assumed, on an interval of [0, 10] ₪. In this case, without reinsurance, the microinsurer needs 10 ₪ at the beginning of each period to ensure its solvency. With reinsurance, the microinsurer would need to secure only the mean cost, 5 ₪, (the reinsurance threshold), because the reinsurer bears all costs above it, plus the premium. The mean value of benefits the reinsurer must pay a microinsurer i is 1.25 ₪ with an SD of 1.61. This has been calculated using the following algorithm[21]:

$$G_{i1}(w) = \begin{cases} 0 & \text{if} \quad w < 0 \\ 0,5 & \text{if} \quad w = 0 \\ \dfrac{2w + 10}{20} & \text{if} \quad 0 < w \le 5 \\ 1 & \text{if} \quad 5 < w \end{cases}$$

which leads us to $\overline{W(1)} = \dfrac{10}{8} = 1.25$ and $\sigma^2_{i1} = \dfrac{5*10^2}{192} = 1.61^2$

The information obtained from the above calculation can now be placed in the equation above, with mean benefit $\overline{W(1)} = 1.25$, and $\sigma_{W1} = 1.61$, to obtain the reinsurance premium. The results are shown in table 7.1.

ANNEX 7C CALCULATING THE MEAN BENEFIT EXPENDITURE AND ITS VARIANCE

To calculate each microinsurer's mean benefit expenditure and its variance, payable at the end of period T, we note:

a(i, t) is the number of individuals covered by microinsurance i at period t.

b(i, t) is the number of benefit types included in the package of microinsurance i during period t.

D(i, j, k, t, .) is the number of occurrences of claims k during the period t for individual j covered by microinsurance i. We assume that D(i, j, k, t, .) is a discrete random variable (whose distribution follows the Poisson law, for example).

E[D(i, j, k, t, .)] is the mean of D(i, j, k, t, .).

Var[D(i, j, k, t, .)] is the variance of D(i, j, k, t, .).

C(i, j, k, t, .) is the amount paid by microinsurance i to individual j each time a claim k is submitted during period t for this individual. We assume that C(i, j, k, t, .) is a random variable.

E[C(i, j, k, t, .)] is the mean of C(i, j, k, t, .).

Var[C(i, j, k, t, .)] is the variance of C(i, j, k, t, .).

T(i, j, k, t, .) is the random variable defined by :

$$T(i, j, k, t, .) = \begin{cases} 0 & \text{if} \quad D(i, j, k, t, .) = 0 \\ C_1(i, j, k, t, .) & \text{if} \quad D(i, j, k, t, .) = 1 \\ C_1(i, j, k, t, .) + C_2(i, j, k, t, .) & \text{if} \quad D(i, j, k, t, .) = 2 \\ \quad \dots & \quad \dots \\ C_1(i, j, k, t, .) + \dots + C_s(i, j, k, t, .) & \text{if} \quad D(i, j, k, t, .) = s \\ \quad \dots & \quad \dots \end{cases}$$

where:

$C_s(i, j, k, t, .) = C(i, j, k, t, .)$ for all s.

It follows that:

$\mathbf{X(i, t, .)} = \sum_j \sum_k \mathbf{T(i, j, k, t, .)}$ (NOTE: this variable is defined in annex 7A.)

$$\overline{X}\,\mathbf{(i, t)} = \sum_j \sum_k \overline{T}\,\mathbf{(i, j, k, t)}$$

Where $\overline{T}$ **(i, j, k, t)** is the mean of T(i, j, k, t, .)
And if the variable T(i, j, k, t, .) is independent, then:

$$\mathbf{Var[X(i, t, .)]} = \sum_j \sum_k \mathbf{var[T(i, j, k, t, .)]}$$

Expression of $\overline{T}$ **(i, j, k, t)**:

$\overline{T}\,\mathbf{(i, j, k, t)} = E[0]Prob[D(i,j,k,t,.)=0]+\dots+E[C_1(i,j,k,t, .) + \dots + C_s(i,j,k,t,.)]$ $Prob[D(i,j,k,t,.) = s]+ \dots$

$\overline{T}$ **(i, j, k, t)** = E[C(i,j,k,t, .)]Prob[D(i,j,k,t,.) = 1] + … + sE[C(i,j,k,t, .)] Prob[D(i,j,k,t,.) = s] + …

$\overline{T}$ **(i, j, k, t)** = E[C(i, j, k, t, .)] **E[C(i, j, k, t, .)]** $\sum_{s=0}^{s=\infty}$ **sProb[D(i, j, k, t, .) = s] =**

E[C(i, j, k, t, .)]E[D(i, j, k, t, .)]

$\overline{T}$ **(i, j, k, t) = E[C(i, j, k, t, .)]E[D(i, j, k, t, .)]**

Thus:

The mean cost paid by microinsurer i to individual j at the end of period t for claim(s) k is equal to the mean cost of claim(s) k paid by microinsurer i during period t multiplied by the mean number of occurrences of claim k during the period t for individual j.

Expression of Var[T(i, j, k, t, .)] :

Var[T(i, j, k, t, .)] = E[T(i, j, k, t, .)^2] – $\overline{T}$ (i, j, k, t)

E[T(i,j,k,t,.)^2]=E[0^2]Prob[D(i,j,k,t,.)=0]+ ... + E{[C_1(i,j,k,t, .) + … + C_s(i,j,k,t,.)]^2} Prob[D(i,j,k,t,.) = s] + …

E[T(i,j,k,t,.)^2] = E(C(i, j, k, t, .)^2) $\sum_{s=0}^{\infty}$ sProb [D(i, j, k, t, .) = s] + 2 [E(C(i, j, k, t, .)^2] $\sum_{s=2}^{\infty} C_s^2$ Prob[D(i, j, k, t, .) = s]

E[T(i,j,k,t,.)^2] = E(C(i, j, k, t, .)^2) E(D(i, j, k, t, .) + 2 [E(C(i, j, k, t, .)^2] $\sum_{s=2}^{\infty} C_s^2$

Prob[D(i, j, k, t, .) = s]

Var[T(i,j,k,t,.)]= E(C(i, j, k, t, .)^2) E(D(i, j, k, t, .) + 2 [E(C(i, j, k, t, .)]^2 $\sum_{s=2}^{\infty} C_s^2$

Prob[D(i, j, k, t, .) = s] – E[C(i,j,k,t,.)]^{2}E[D(i,j,k,t,.)]^2

Var[T(i,j,k,t,.)] = E(C(i, j, k, t, .)^2) E(D(i, j, k, t, .) + E[C(i,j,k,t,.)]^2{2 $\sum_{s=2}^{\infty} C_s^2$

Prob[D(i, j, k, t, .) = s] – E(D(i, j, k, t, .)^2}

Var[T(i,j,k,t,.)]= E(C(i, j, k, t, .)^2) E(D(i, j, k, t, .) + E[C(i,j,k,t,.)]^2{ $\sum_{s=2}^{\infty}$ s(s – 1)

Prob[D(i, j, k, t, .) = s] – E(D(i, j, k, t, .)^2}

But:

$$\sum_{s=2}^{\infty} s(s-1)\ \text{Prob}[D(i, j, k, t, .) = s] = \sum_{s=2}^{\infty} s^2\ \text{Prob}[D(i, j, k, t, .) = s] - \sum_{s=2}^{\infty} s\ \text{Prob}[D(i, j, k, t, .) = s]$$

$$\sum_{s=2}^{\infty} s^2\ \text{Prob}[D(i, j, k, t, .) = s] = E(D(i, j, k, t, .)^2) - \text{Prob}[D(i, j, k, t, .) = 1]$$

$$\sum_{s=2}^{\infty} s\text{Prob}[D(i, j, k, t, .) = s] = E(D(i, j, k, t, .)) - \text{Prob}[D(i, j, k, t, .) = 1]$$

Therefore:

$$\sum_{s=2}^{\infty} s(s-1)\text{Prob}[D(i, j, k, t, .) = s] = E(D(i, j, k, t, .)^2) - E(D(i, j, k, t, .))$$

And:

$$\text{Var}[T(i,j,k,t,.)] = E[D(i,j,k,t,.)]E[C(i,j,k,t,.)^2] + E[C(i,j,k,t,.)]^2\{E[D(i,j,k,t,.)^2] - E[D(i,j,k,t,.)^2] - E[D(i,j,k,t,.)]\}$$

$$\text{Var}[T(i,j,k,t,.)] = E[D(i,j,k,t,.)]E[C(i,j,k,t,.)^2] + E[C(i,j,k,t,.)]^2\{\text{Var}[D(i,j,k,t,.)] - E[D(i,j,k,t,.)]\}$$

$$\text{Var}[T(i,j,k,t,.)] = E[D(i,j,k,t,.)]\text{Var}[C(i,j,k,t,.)] + E[C(i,j,k,t,.)]^2\text{Var}[D(i,j,k,t,.)]$$

Thus:

The variance of the cost paid by microinsurer i to individual j at the end of period t for claim k is equal to the sum of the variance of the cost of claim k paid by microinsurance i during period t multiplied by the mean number of occurrences of claim k during period t for individual j belonging microinsurer i, plus the square root of the mean cost of claim k paid by microinsurer i during period t multiplied by the variance of the number of occurrences of claim k during the period t for individual j.

Example:

If distributions of D(i, j, k, t, .) and C(i, j, k, t, .) are given by these tables :

D(i, j, k, t, .)	**0**	**1**	**2**	**3**	**4**
Probability	0.2	0;2	0.2	0.2	0.2

C(i, j, k, t, .)	**1**	**2**
Probability	0.25	0.75

Then:

E[D(i,j,k,t,.)] = 2 and Var[D(i,j,k,t,.)] = 2

E[C(i,j,k,t,.)] = 1.75 and Var[C(i,j,k,t,.)] = 0.1875

Thus:

E[T(i,j,k,t,.)] = $2_*1.75 = 3.5$ and Var[T(i,j,k,t,.)] = $2_*0.1875 + 1.75^2{}_*2 = 6.5$

In this case it easy to compute the distribution of T(i,j,k,t,.):

T(i,j,k,t,.)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Probability	256/ 1280 = 20%	64/ 1280 = 5%	208/ 1280 = 16.25%	100/ 1280 = 7.81%	181/ 1280 = 14.14%	120/ 1280 = 9.38%	162/ 1280 = 12.66%	108/ 1280 = 8.44%	81/ 1280 = 6.32%

And it is easy also to compute the mean and the variance of T(i,j,k,t,.):

E[T(i,j,k,t,.)] = 3.5 and Var[T(i,j,k,t,.)] = 6.5

Therefore:

$$\overline{X}(i, t) = \sum_{j=1}^{a(i, t,)} \sum_{k=1}^{b(i, t,)} E[D(i, j, k, t, .)]\, E[C(i, j, k, t, .)]$$

and

$$Var[x(I, t, .)] = \sum_{j=1}^{a(i, t,)} \sum_{k=1}^{b(i, t,)} \{E[D(i,j,k,t,.)]Var[C(i,j,k,t,.)] + E[C(i,j,k,t,.)]^2 Var[D(i,j,k,t,.)]\}$$

Particular case:

If we assume that occurrences of each claim are independent of individuals and that costs are also independent of individuals, we have:

D(i, j, k, t, .) = D(i, k, t, .) and C(i, j, k, t, .) = C(i, k, t, .) or all j and k.

In this case we have:

$$\overline{X}(i, t) = a(i, t) \sum_{k=1}^{b(i, t,)} E[D(i, t, .)\, E[C(i, t, .)]$$

and

$$Var[X(i, t, .)] = a(i,t) \sum_{k=1}^{b(i, t)} \{E[D(i,j,k,t,.)]Var[C(i,j,k,t,.)] + E[C(i,j,k,t,.)]^2 Var[D(i,j,k,t,.)]\}$$

ANNEX 7D CALCULATING THE EFFECTS OF REINSURANCE

Using the previous example as the basis for an analysis of the impact of reinsurance over time, and assuming that all parameters remain unchanged (distribution probability of microinsurers' balance sheets, the number of reinsured microinsurers, and 5 percent limit on the reinsurer's insolvency risk), the central limit theorem can now be applied. Thus, if the reinsurance contract extends over a number of periods T, the premium Δ paid by each microinsurer for each period must equal:

$$\Delta = \overline{W} - \frac{A(n, T)}{nT} + 1.65\sigma_W / \sqrt{nT}$$

Where:

$\overline{W}$ is the mean of random variable W equal to benefits paid by the reinsurer each period,

σ_W is the SD of random variable W,

n is the number of microinsurers,

T is the duration of the reinsurance contract,

A(n,T) is the reinsurer's initial capital, decreased by its managing costs for periods T.

This example can be extended to a more general case, where

$A(n, T) = B_0 - Ts(n)$

where B_0 is the reinsurer's initial capital and we assume that, for each period, administrative costs s(n) depend only upon the number of reinsured units and that initial seed capital will be depleted over time. As this capital is depleted, the reinsurance premium will increase. By the same token, when the reinsurer accumulates surpluses during the same period, it can decrease the required premium. The general mathematical expression of the condition when the premium is likely to decrease over time is that this situation will occur when the derivative of Δ as a function of time is negative. The results of this derivation are the following:

For any given n, if $T \geq \frac{1.47B_0^2}{n\sigma_W^2}$, then premium Δ decreases as the duration T of the contract increases.

This relationship implies that the larger the number of microinsurers in the pool (n), and the greater the variance of the reinsurer's benefit expenditure (σ_W), the shorter the time from the initiation of the contract to the point when the premium will start to decrease.[22]

Simulated results, corroborating that a larger number of pooled microinsurers can reduce the premium, and that the impact of a larger pool tapers off at a certain size, are provided in figure 7.4.

NOTES

1. A review of the actuarial literature reveals the existence of numerous reinsurance models, which differ in the content and complexity of the benefit package (Outreville, chapter 3, this volume). Various simulation software packages have been developed

to assess the impact of these benefit packages in specific settings (Geneva Association 1982; Brown and Galitz 1983, 1984). However, these models and software were designed for use in the economic context of industrial countries and have not been effective when used in the context of microinsurance units in developing countries.

2. This variance is a function of group size; the ratio between the variance and the expected mean decreases as group size increases (Dror 2001).
3. The large number of microinsurance units is needed to apply the law of large numbers.
4. The assumption of uniform distribution is valid when there is no reliable information on the risk probability and on the variance of benefit cost. Any alternative assumption about distribution would probably have given a lower premium.
5. To avoid confusion between the acronyms MU (monetary unit) and MIU (microinsurance unit), the symbol ₪ (stylized MU) is used here, without designating any specific currency.
6. Under this distribution law, the variance is equal to the mean.
7. Under this distribution law, the variance is equal to twice the mean.
8. The *Social Re Data Template* is described in appendix A, this volume. The data needs are discussed in detail in chapter 16, this volume.
9. This is a result of the complexity of the probability laws for microinsurers' business results and of the complex analytical expression predicting the probability of the reinsurer's bankruptcy (a convolution product of truncated laws).
10. A Poisson law was used, as is usual in such a case; the parameter used is the mean of each cost-generating event within the population.
11. CHI2 law was used for simplicity, but any other law would do.
12. The business result is a function of the distribution of cost-generating events it has to pay and the distribution of unit costs that apply to these events.
13. Monte Carlo simulation consists of generating pseudo random numbers following a given probability distribution with a view to obtaining empirically the probable distributions of random variables (see also glossary entry for this term at end of this volume).
14. This assumes that the microinsurer's members pay all their contributions in full and on time.
15. (0.5*35.70)/75 = 23.8.
16. In relative terms, all microinsurers can expect the discretionary budget to be around 80 percent of premiums paid.
17. n = 500, P = 1 percent, unit cost = 15, average total cost = 75, SD = 35.70.
18. The role of subsidies in achieving full recovery rate is discussed in Busse, chapter 13, this volume.
19. In this context, *catastrophic risks* generate higher expenses than the worst-case scenario predictable by the applicable statistical laws. Such cases can occur through such events as epidemics affecting the whole community, acts of nature, and the like.
20. The analytical calculation of the reinsurance premium is often impossible because of the complexity of the probability laws for microinsurers' business results and the complex analytical expression predicting the probability of the reinsurer's bankruptcy (a convolution product of truncated laws).

21. Annex 7A explains the principles underlying the algorithm.

22. For any given T, if 0,825 $\sigma_W\sqrt{nT} - T[ns'(n) - s(n)] \geq B_0$, then premium Δ decreases as the number of reinsured microinsurers n increases (s'(n) designates the first derivative of s).

REFERENCES

Bass, J. 1974. *Elément de calcul des probabilités*. 3d ed. Paris: Masson.

Brown, Z.M., and L. Galitz. 1983. *The ASIR System*. Working Paper No. 66. Geneva: Geneva Association.

_____. 1984. *The ASIR System*. Working Paper No. 76. Geneva: Geneva Association.

Dror, D.M. 2001. "Reinsurance of Health Insurance for the Informal Sector." *World Health Organization Bulletin* 79(7):672–78.

Geneva Association. 1982. "The ASIR Model." *The Geneva Papers on Risk and Insurance* 7 (October 25):279–392.

Lesage, J.J. 1999. *Modélisation des systèmes réactifs*. Paris: Hermès Ed.

Monsef, Y. 1997. "Modeling and Simulation of Complex Systems Concepts." In E. Kerckhoffs, H. Pierreval, R.N. Zobel, and A. Lehmann, eds., *Frontiers in Simulation*. Society for Computer Simulation. Manchester: Department of Computer Science, University of Manchester.